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Note on an Unreliable Quorum Queueing System with Random Server Capacity

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Abstract: We consider a bulk-service queueing system with random server capacity where the server is subject to random breakdowns. Start and end of idle periods are regulated by an accumulation level r. If the queue is larger than or equal to r when a service ends, the server takes a group of r customers to serve. On the other hand, if the queue is smaller than r when a service ends, the server remains idle and waits for the queue to reach r, then takes a group of at most r customers to serve. We derive the probability generating function of the state probabilities in the steady-state at a service completion instant and at an arbitrary instant of time. We also derive various performance measures. A detailed numerical example is provided.

Keywords: Bulk service, Markov chain, queue, random capacity, semi-regenerative process.

1. Introduction

Queueing systems with unreliable server, or where the server is subject to random breakdowns, have been heavily considered by researchers in the last few years. A large amount of models are available and more and more models are being studied. It is not uncommon to see scholars derive an optimal management policy for such systems. For example, Wang [16] considers the M/G/1 queue with second optional service and server breakdowns. Wang et al. consider the optimal control of the M/G/1 queueing system with server breakdowns and general startup times under N-policy [18] and under T-policy [17]. Ke et al. [10] deal with the optimal (d, c) vacation policy for a finite buffer M/M/c queue with unreliable servers and repairs. Another recent paper is that of Paz and Yechiali [13] who study an M/M/1 queue in random environment with disasters, assuming disasters are more general notion than server breakdown. Instead of reviewing all the research on this topic, we refer the reader to the recent survey of Krishnamoorthy et al. [11].

We also consider in this paper a queueing system with an unreliable server. It is a bulk service system and the server does not start service until the number of customers in the queue accumulates to a fixed level. Such systems were called quorum systems by Chaudhry and Templeton [3]. They were also called queues with fixed accumulation level (Dshalalow and Tadj [7]), service delayed queueing systems (Abolnikov and Dshalalow [1]), systems under q-policy (Dshalalow [6]), etc. For extensive discussions of such systems and their analysis see for example, Medhi [12] and Dshalalow [6].

Another characteristic of the queueing system under consideration is that the capacity of the server becomes random following an idle period. These systems were introduced and studied by Dshalalow and Tadj [7-9] who also provide some practical applications.

We further assume that breakdowns, services, and repairs depend on the number of customers being served. Breakdowns occur according to a Poisson process, but service times and repair times follow general probability distributions. Most papers available in the literature use the supplementary variable technique to analyze their model. Following a description of the model in Section 2, we

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combine in Section 3 the embedded Markov chain with semi-regenerative techniques to obtain the probability generating function (PGF) of the system state probabilities in the steady-state. We also derive performance measures in Section 4, present a detailed numerical example in Section 5, and conclude the paper in Section 6.

2. Model Description

We consider a single-server queueing system with an infinite waiting room. The arrival process is a Poisson process with positive rate λ. The service is in bulk and the process is controlled through a threshold r as follows: When the queue size is larger than or equal to r, service is provided to a group of exact r customers. When the queue size is smaller than r, service is provided to a group of size at most r customers.

More formally, let $Q(t)$ denote the number of customers in the system at any time t, T_n represent the completion time of the nth service, and $Q_n = Q(T_n^+)$ the number of customers in the system at the nth service completion instant.

When $Q_n \geq r$, the server uses its whole capacity and starts serving a group of r customers, all at once. The service times are i.i.d random variables with a general probability distribution $B(t)$, and the average service time $b^{(1)}$ and the second moment $b^{(2)}$ are finite. While the server is up and working, it is subject to breakdowns according to a Poisson process with rate $\alpha > 0$. When the server breaks down the service must stop until the repair is completed and the service time of this group of customers is still valid, it is repaired immediately to return to the new state and operate. The repair time follows a general distribution function $R(t)$, and the average repair time $r^{(1)}$ and the second moment $r^{(2)}$ are finite. Now, we introduce the modified service, which is the time required to process a batch of customers including any breakdowns and repairs. The modified service time has distribution function $G(t)$, and the average modified service time and the second moment are denoted by $g^{(1)}$ and $g^{(2)}$. It is clear that both $g^{(1)}$ and $g^{(2)}$ are finite. Indeed, the Laplace-Stieljjes Transform (LST) $G^*(\theta)$ of the modified service, the LST $B^*(\theta)$ of the actual service time, and the LST $R^*(\theta)$ of the repair time are related by, see Tang [14, 15],

$$G^*(\theta) = B^*\left(\theta + \alpha\left(1 - R^*(\theta)\right)\right),$$

so that

$$g^{(1)} = b^{(1)}\left(1 + \alpha r^{(1)}\right),$$

and

$$g^{(2)} = b^{(2)}\left(1 + \alpha r^{(1)}\right)^2 + \alpha b^{(1)} r^{(2)},$$

are finite.

In the other case, when $Q_n < r$, the server remains idle, waiting for the queue size to reach the threshold level r. However, by the time the queue size becomes r, the server capacity may have changed, so that only a random number of customers c_n are served all at once. When j $(1 \leq j \leq r)$ customers are selected to receive service, the service times are i.i.d random variables with a general probability distribution $B_j(t)$, and the average service time $b_j^{(1)}$ and the second moment $b_j^{(2)}$ are finite. The random server capacity c_n are i.i.d random variables with probability mass function $(\gamma_1, \gamma_2, \cdots, \gamma_r)$. Also, while the server is up and working, it is subject to breakdowns according to a Poisson process with rate $\alpha_j > 0$. When the server breaks down the service must stop until the repair is completed and the service time of this group of customer is still valid, it is repaired immediately to return to the new state and operate. The repair time follows a general distribution function $R_j(t)$, and the average repair time $r_j^{(1)}$ and the second moment $r_j^{(2)}$ are finite. Therefore, breakdowns are Poisson with rate α_j, dependent on the number j of customers being served. Service and repairs also depend on j. Now, we introduce the modified service, which is the time required to process a batch of customers including any breakdowns and repairs. The modified service time has distribution function $G_j(t)$, and the average modified service time and the second moment are denoted by $g_j^{(1)}$ and $g_j^{(2)}$. It is clear that both $g_j^{(1)}$ and $g_j^{(2)}$ are finite. Indeed, the LST $G_j^*(\theta)$ of the modified service, the LST $B_j^*(\theta)$ of the actual service time, and the LST $R_j^*(\theta)$ of the repair time are related by

$$G_j^*(\theta) = G_j^*\left(\theta + \alpha_j\left(1 - R_j^*(\theta)\right)\right), \qquad j = 1, \cdots, r,$$

so that

$$g_j^{(1)} = b_j^{(1)}\left(1 + \alpha_j r_j^{(1)}\right), \qquad j = 1, \cdots, r,$$

and

$$g_j^{(2)} = b_j^{(2)}\left(1 + \alpha_j r_j^{(1)}\right)^2 + \alpha_j b_j^{(1)} r_j^{(2)}, \qquad j = 1, \cdots, r,$$

are finite.

We assume that the arrival, service, breakdowns, and repair processes are independent of each other. We will denote by P^i the conditional probability given $Q_n = i$, while E^i will denote the corresponding expectation.

3. Model Analysis

The stochastic process $\{Q_n, n = 0,1,\cdots\}$ is a Markov chain since it satisfies the following recursive formula:

$$Q_{n+1} = \begin{cases} r - c_{n+1} + V_{n+1}, & Q_n < r, \\ Q_n - r + V_{n+1}, & Q_n \geq r. \end{cases} \tag{1}$$

Recall that c_n represents the number of customers served in a single batch following an idle period. The number of customer arrivals during the nth modified service time is denoted by V_n and is such that $E[z^{V_{n+1}}] = G^*(\lambda - \lambda z)$.

3.1. Ergodicity

Let $\mathcal{A} = (p_{ij})$ denote the transition probability matrix (TPM) of $\{Q_n, n = 0,1,\cdots\}$. Then the first r rows (row 0 to row $(r-1)$) are all identical, while the rest of the elements form an upper triangular matrix. Thus, $\mathcal{A}$ is a Δ-matrix, see Abolnikov and Dukhovny [2], and stability of the system is insured provided

$$\left.\frac{d}{dz}A_r(z)\right|_{z=1} < r,$$

where $A_r(z)$ is the PGF of the rth row of the TPM $\mathcal{A}$. It is readily seen that the above condition is equivalent to

$$\rho < 1,$$

where

$$\rho = \frac{\lambda b^{(1)}\left(1 + \alpha r^{(1)}\right)}{r}.$$

3.2. PGF at a service completion epoch

Assuming now the above condition is satisfied so that the Markov chain is ergodic, let us introduce the limiting state probabilities $p_i = \lim_{n\to\infty} P(Q_n = i)$. Also, let $P = (p_0, p_1, \cdots)$ and $p(z) = \sum_{i=0}^{\infty} p_i z^i$.

Let $A_i(z) = E[z^{Q_{n+1}} \mid Q_n = i]$. Then obviously $A_i(z) = \sum_{j=0}^{\infty} p_{ij} z^j$ is the PGF of the ith row of the TPM $\mathcal{A}$. Since $\{Q_n, n = 0,1,\cdots\}$ is ergodic, then $P(z) = \lim_{n\to\infty} E^i[z^{Q_{n+1}}]$, where $E^i[z^{Q_{n+1}}] = E^i[E[z^{Q_{n+1}}|Q_n]] = E^i\left[A_{Q_n}(z)\right] = \sum_{j=0}^{\infty} A_j(z) P^i(Q_n = j)$. Therefore

$p(z)=\sum_{i=0}^{\infty}A_i(z)p_i$. From (1), we have

$$A_i(z)=\begin{cases}\sum_{k=1}^{r} z^{r-k}G_k^*(\lambda-\lambda z)\gamma_k, & i<r,\\ z^{i-r}G^*(\lambda-\lambda z), & i\ge r,\end{cases}$$

and it follows that

$$p(z)=\frac{\sum_{i=0}^{r-1}\left[z^rA(z)-z^iG^*(\lambda-\lambda z)\right]p_i}{z^r-G^*(\lambda-\lambda z)}.$$

When $i<r$, $A_i(z)$ is independent of i and thus has been denoted by $A(z)$. The probabilities $p_0,\cdots,p_{r-1}$ are obtained by solving the linear system of r equations in r unknowns,

$$\sum_{i=0}^{r-1}\frac{d^k}{dz^k}\left[A(z)-z^i\right]\Bigg|_{z=z_s} = 0, \qquad k=0,\cdots,k_s-1;\ s=1,\cdots,S,$$

$$\sum_{i=0}^{r-1}\left\{\varepsilon+r-i+\lambda\left(\sum_{k=1}^{r}g_k^{(1)}\gamma_s-g^{(1)}\right)\right\}p_i = r-\lambda g^{(1)},$$

where $\varepsilon=r-E[c_n]$ and z_s are the roots of the characteristics equation,

$$z^r-G^*(\lambda-\lambda z)=0,$$

in the region $\bar{B}(0,1)\backslash\{1\}$ with their multiplicities k_s such that $\sum_{s=1}^{S}k_s=r-1$, see Chaudhry and Templeton [3] and Chaudhry et al. [4].

3.3. PGF at an arbitrary instant of time

Assuming now the above condition is satisfied so that the semi-regenerative process $\{Q(t),t\ge 0\}$ is ergodic, let us introduce the limiting state probabilities $\pi_i=\lim_{t\to\infty}P(Q(t)=i)$. Also, let $\pi(z)=\sum_{i=0}^{\infty}\pi_iz^i$. Then, using the main convergence theorem for semi-regenerative processes, we have, see Çinlar [5],

$$\lambda(1-z)\,P\beta\pi(z)=(1-z^r)p(z)-\left[\sum_{k=1}^{r}(1-z^{r-k})G_k^*(\lambda-\lambda z)\right]\left[\sum_{i=0}^{r-1}p_i\right].$$

$P\beta$ is called the stationary mean service cycle and will be derived in the next section.

4. System Performance Measures

Using the PGF's of the previous section, various performance measures can be obtained.

4.1. Stationary mean service cycle

First we calculate the stationary mean value of the service cycle $P\beta$ to conclude the evaluation of the PGF $\pi(z)$. Let $\beta_i=E^i[T_{n+1}-T_n]$ and $\beta=(\beta_0,\beta_1,\cdots)^T$. Then,

$$\beta_i=\begin{cases}\frac{r-i}{\lambda}+\sum_{k=1}^{r}g_k^{(1)}\gamma_k, & i<r,\\ g^{(1)}, & i\ge r,\end{cases}$$

so that

$$P\beta=\sum_{i=0}^{r-1}\left(\frac{r-i}{\lambda}+\sum_{k=1}^{r}g_k^{(1)}\gamma_k-g^{(1)}\right)p_i+g^{(1)}.$$

4.2. Stationary mean server load

Note that the server capacity is given by

$$\Gamma_{n+1} = \begin{cases} c_{n+1}, & Q_n < r, \\ r, & Q_n \geq r. \end{cases}$$

The stationary mean value of the server load ℓ defined by

$$\ell = \lim_{n\to\infty} E^{Q_n}[\Gamma_n],$$

can be shown to be equivalent to

$$\ell = r - \varepsilon \sum_{i=0}^{r-1} p_i.$$

4.3. Stationary system intensity

Also known as the offered load, it is defined by

$$\mathfrak{I} = \frac{P\beta}{\frac{1}{\lambda}}.$$

In this queueing system, it can be evaluated by

$$\mathfrak{I} = \sum_{i=0}^{r-1} \left[r - i + \lambda \left(\sum_{k=1}^{r} g_k^{(1)} \gamma_k - g^{(1)} \right) \right] p_i + \lambda g^{(1)}.$$

Also, using the summability-to-one condition $P(1) = 1$, it can be shown that $\mathfrak{I} = \ell$.

4.4. Mean system size at a service completion epoch

Using the PGF $p(z)$, we find the mean system size at a service completion epoch $L_d = p'(z)|_{z=1}$ as

$$L_d = \frac{N''(1) - D''(1)}{2D'(1)},$$

where

$$N''(1) = \sum_{i=0}^{r-1} \left[2r(r-1) + 2r \left(\varepsilon + \lambda \sum_{k=1}^{r} g_k^{(1)} \gamma_k \right) + (2r+1)E[c] - E[c^2] \right.$$
$$\left. + \ 2\lambda \left(\sum_{k=1}^{r} g_k^{(1)} \gamma_k - \sum_{k=1}^{r} k g_k^{(1)} \gamma_k - i g^{(1)} \right) - i(i-1) + \lambda^2 \left(\sum_{k=1}^{r} g_k^{(2)} \gamma_k - g^{(2)} \right) \right] p_i,$$

$$D''(1) = r(r-1) - \lambda^2 g^{(2)}, \qquad D'(1) = r - \lambda g^{(1)}.$$

4.5. Mean system size at an arbitrary instant of time

Using the PGF $\pi(z)$, we find the mean system size at an arbitrary instant of time $L_c = \pi'(z)|_{z=1}$ as

$$L_c = \frac{1}{\ell} \left\{ \frac{r(r-1)}{2} + rL_d - \sum_{s=1}^{r} (r-s)\gamma_s \left[\frac{r-s-1}{2} + \lambda g_s^{(1)} \right] \left[\sum_{i=0}^{r-1} p_i \right] \right\}.$$

4.6. Mean idle period

Since the probability that the server is idle in the equilibrium is given by

$$\frac{\sum_{i=0}^{r-1} p_i e_{\lambda,r-i}(t)}{\sum_{i=0}^{r-1} p_i},$$

where $e_{\lambda,r-i}(t)$ represents the k-Erlang probability density function with parameter λ, it follows that the mean idle period in the steady-state is given by

$$I = \frac{\sum_{i=0}^{r-1} p_i \frac{r-i}{\lambda}}{\sum_{i=0}^{r-1} p_i}.$$

4.7. Mean busy period

The proportion of time the server is idle in the steady-state $\zeta = \sum_{i=0}^{r-1} p_i$ is given by

$$\zeta = \frac{I}{I+B}.$$

Therefore, the mean busy period can be calculated from

$$\mathrm{B} = \frac{1-\zeta}{\zeta}\mathrm{I}.$$

4.8. Mean busy cycle

Finally, the mean busy cycle, which is the sum of the mean idle period and the mean busy period, is found to be

$$C = \frac{1}{\zeta} I.$$

5. Numerical Example

Assume customers arrive to a service facility at a rate $\lambda = 0.6$ and the server is subject to Poisson failures with rate $\alpha = 0.05$. For illustration, assume $r = 2$. When the queue size is greater than or equal to r, the server serves a group of 2 customers according to the exponential distribution with mean$b^{(1)} = 2.7$. Following a breakdown, the server is repaired according to an exponential distribution with mean $r^{(1)} = 1.3$. Note that $b^{(2)} = 2\left[b^{(1)}\right]^2 = 14.58$ and $r^{(2)} = 2\left[r^{(1)}\right]^2 = 3.38$. We calculate $g^{(1)} = b^{(1)}\left(1+\alpha r^{(1)}\right) = 2.8755$ and $g_j^{(2)} = b_j^{(2)}\left(1+\alpha_j r_j^{(1)}\right)^2 + \alpha_j b_j^{(1)} r_j^{(2)} = 16.9933$. A steady-state exists since $\rho = \frac{\lambda g^{(1)}}{r} = 0.8627$ is smaller than 1.

The characteristic equation reduces to

$$z^2 - \frac{1+\lambda r^{(1)}(1-z)}{1+\lambda r^{(1)}(1-z)+\lambda b^{(1)}(1-z)[1+\alpha r^{(1)}+\lambda r^{(1)}(1-z)]} = 0,$$

or simply

$$-\lambda^2 b^{(1)} r^{(1)} z^3 + \lambda\left[r^{(1)} + b^{(1)}\left(1+\alpha r^{(1)}+\lambda r^{(1)}\right)\right]z^2 - z - 1 - \lambda r^{(1)} = 0.$$

This equation has the three roots 2.4126, 1.1005, and $z_1 = -0.5305$ which is inside the open unit ball.

When the queue size is smaller $r = 2$, i.e., there is 0 or 1 customer waiting, the server remains idle and waits for the queue to have 2 customers. Then he serves a group of either 1 customer with probability $\gamma_1 = 0.3$ or 2 customers with probability $\gamma_2 = 1-\gamma_1$, according to the exponential distribution with respective means $b_1^{(1)} = b^{(1)}$ and $b_2^{(1)} = \frac{b^{(1)}}{2}$. In each case, following breakdowns with respective rates $\alpha_1 = \alpha$ and $\alpha_2 = \frac{\alpha}{2}$, the server is repaired according to an exponential distribution with respective means $r_1^{(1)} = r^{(1)}$ and $r_2^{(1)} = \frac{r^{(1)}}{2}$. Note that $b_1^{(2)} = 2\left[b_1^{(1)}\right]^2 = 14.58, b_2^{(2)} = 2\left[b_2^{(1)}\right]^2 =$

$3.645, r_1^{(2)} = 2\left[r_1^{(1)}\right]^2 = 3.38$ and $r_2^{(2)} = 2\left[r_2^{(1)}\right]^2 = 0.845$. We calculate $g_1^{(1)} = b_1^{(1)}\left(1 + \alpha_1 r_1^{(1)}\right) = 2.8755, g_1^{(2)} = b_1^{(2)}\left(1 + \alpha_1 r_1^{(1)}\right)^2 + \alpha_1 b_1^{(1)} r_1^{(2)} = 16.9933$, and $g_2^{(1)} = b_2^{(1)}\left(1 + \alpha_2 r_2^{(1)}\right) = 1.3719$, $g_2^{(2)} = b_2^{(2)}\left(1 + \alpha_2 r_2^{(1)}\right)^2 + \alpha_2 b_2^{(1)} r_2^{(2)} = 3.7929$.

To obtain p_0 and p_1, we solve the linear system of two equations in two unknowns

$$[A(z_1) - 1]p_0 + [A(z_1) - z_1]p_1 = 0,$$

$$\left[\varepsilon + 2 + \lambda\left(\sum_{k=1}^{2} g_k^{(1)}\gamma_k - g^{(1)}\right)\right]p_0 + \left[\varepsilon + 1 + \lambda\left(\sum_{k=1}^{2} g_k^{(1)}\gamma_k - g^{(1)}\right)\right]p_1 = 2 - \lambda g^{(1)}.$$

Here $A(z) = zG_1^*(\lambda - \lambda z)\gamma_1 + G_2^*(\lambda - \lambda z)\gamma_2$, $E[C] = \gamma_1 + 2\gamma_2 = 1.7$ and $\varepsilon = r - E[c_n] = 0.3$. We obtain $p_0 = 0.1202$ and $p_1 = 0.1108$. We also obtain $\pi_0 = 0.0623$ and $\pi_1 = 0.1193$ and calculate all the performance measures: $P\beta = 3.2178$, $\Im = \ell = 1.9307$, $L_d = 39.4241$, $L_c = 41.2955$, $I = 2.5340$, $B = 11.3929$, and $C = 13.9269$.

If breakdowns happen too often, we would expect the number of customers in the system to grow higher and higher. This is indeed the case, as we increased the breakdown rate α from 0.01 to 0.10 by increments of 0.01 and recorded the mean system size at a service completion epoch and at an arbitrary epoch. The variations of these two measures are represented in Figure 1.

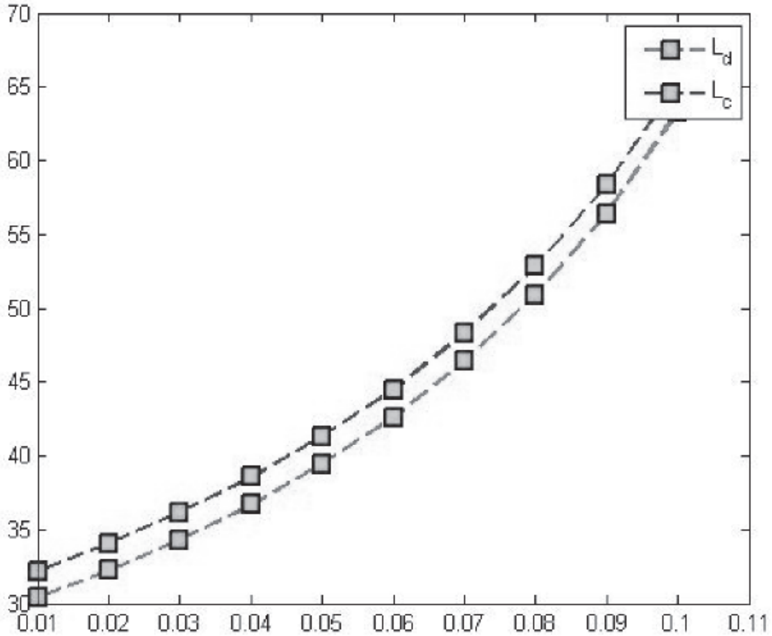

Figure 1. Effect of the breakdown rate α on the mean system sizes L_d and L_c.

6. Conclusion

We have considered in this paper a bulk service queueing system where the server capacity becomes random following an idle period. Customers arrivals and server breakdowns occur according to Poisson processes, but all other processes follow general distributions. A kind of state dependence is assumed as the breakdown, service, and repair processes all depend on the number of customers being served simultaneously. The PGF of the system size in the steady-state is obtained. Since all the main performance measures are obtained, it is possible to design an optimal management policy for this system where the threshold level r would be the control variable.

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